November 4 Finite Geometries and Axiomatic Systems

Three Point Geometry | Four Point Geometry | Five Point Geometry | Four Line Geometry
Fano's Geometry | Not Categorical Axiomatic System | Worksheet

An axiomatic system has:

a) undefined terms

b) a set of axioms which are independent and consistent

An axiomatic system is categorical if there is basically only one model for it (names and objects, of course, may change).

An axiomatic system is complete if the addition of a new axiom is redundant.

A categorical axiomatic system is complete!!!!!

Three Point Geometry

Axioms for the Three Point Geometry

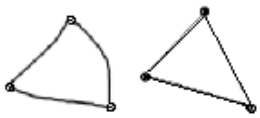
1. There exist exactly three distinct points.

2. Each two distinct points are on exactly one line.

3. Not all the points are on the same line.

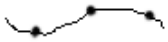
4. Each two distinct lines are on at least one point.

Models



NOTE: Nothing is said about the type of line - whether it is straight or curved.

NOT A MODEL



Theorems

1. Each two distinct lines are on exactly one point.

2. There exist exactly three lines.

Four Point Geometry

Axioms for the Four Point Geometry

1. There exist exactly four points.

2. Each two distinct points have exactly one line that contains both of them

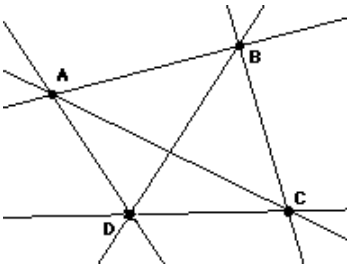
Model

3. Each line is on exactly two points.

Theorems

1. The four point geometry has exactly 6 lines.

2) Each point of the geometry has exactly 3 lines on it.



NOTE: No point exists where $\overleftrightarrow{AC}$ and $\overleftrightarrow{BD}$ seem to intersect.

Five Point Geometry

Axioms for the Five Point Geometry

1. There exist exactly five points.

2. Each two distinct points have exactly one line on both of them.

3. Each line has exactly two points.

Theorems

1. The five point geometry has exactly 10 lines.

2. Each point of the geometry has exactly 4 lines on it.

Model

Four line Geometry

Axioms for the Four Line Geometry MODEL

1. There exist exactly four lines.

2. Any two distinct lines have exactly one point on both of them.

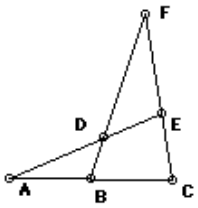
3. Each point is on exactly two lines.

Theorems

1. There exist exactly 6 points

2. Each line has exactly 3 points on it.

Models



NOTE: A finite geometry can also sometimes be represented by a table. Here the columns show a line and each capital letter is a point.

A	A	B	C
B	D	D	E
C	E	F	F

Fano's Geometry - 1892

Axioms for Fano's Line Geometry

1. There exist at least one line.

2. Every line of the geometry has exactly three points on it.

3. Not all the points of the geometry are on the same line.

4. For two distinct points, there exists exactly one line on both of them.

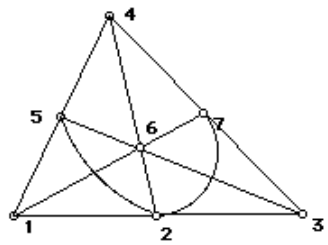
5. Each two lines have at least one point on both of them

Theorems

1. Each two distinct lines have exactly one point in common.

2. There exist exactly 7 points and 7 lines.

Model



An interesting model for Fano's Geometry is the Committee model where each line would represent a committee and and each point would represent a person on that committee.

Committee 1	Committee 2	Committee 3	Committee 4
Ann	Ann	Ann	Bob
Bob	Darla	Fran	Darla
Cary	Elle	George	Fran
Committee 5	**Committee 6**	**Committee 7**	
Cary	Bob	Cary	
Elle	Elle	Darla	
Fran	George	George	

An Example of an axiomatic system that is not categorical

Axioms

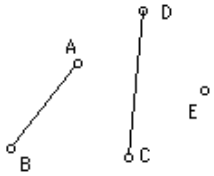
1. There exist five points

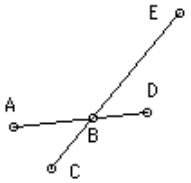
Two Completely Different Models

2. Each line is a subset of those five points.

3. There exist two lines

4. Each line contain at least two points.





from Modern Geometries
by James Smart
pp.12-24 and 34 - 35.

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