Algebra rules

A^3–Assess, Analyze, and Address
Summer 2017

Algebraically solve for x,

$$x + 1 = 2$$

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$$x + 1 = 2$$

$$x + 1 + (-1) = 2 + (-1)$$

$$x + (1 + -1) = 1$$

$$x + 0 = 1$$

$$x = 1$$

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$$x + 1 + (-1) = 2 + (-1)$$

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$$x + 0 = 1$$

$$x = 1$$

additive identity

Identity element

- The identity element *E* for an operation ♥ is the unique element such that

$$a \heartsuit E = E \heartsuit a = a$$

For every element a.

- What operations (+, -, *, /, ∘) have an identity element?

Identity element

- Addition: $0 + a = a + 0 = a$
- Multiplication: $1 * a = a * 1 = a$
- Composition: $(g \circ f)(x) = (f \circ g)(x) = x$, where $f(x) = x$
- Not subtraction
- Not division

Why important?

- Useful for solving equations
- Useful for proofs, e.g, derivative product rule
- Multiply by reciprocal? Why bother …

$$\frac{\frac{1}{2}}{\frac{3}{4}} = \frac{\frac{1}{2}}{\frac{3}{4}} \cdot \frac{\frac{4}{3}}{\frac{4}{3}} = \frac{1}{2} \cdot \frac{4}{3}$$

Are fractions the gatekeeper for algebraic reasoning?

Graham Fletchy

Algebraically solve for x,

$$x + 1 = 2$$

$$x + 1 + (-1) = 2 + (-1)$$

$$x + (1 + -1) = 1$$

$$x + 0 = 1$$

$$x = 1$$

additive inverse

Inverse element

- For operation ♥, an inverse of any element a is the element a^{-1} so that,

$$a \heartsuit a^{-1} = a^{-1} \heartsuit a = E$$

Where E is the identity element.

- What operations (+, -, *, /, ◦) have an inverse elements?

Inverse elements

- Addition: a, -a
- Multiplication: a, 1/a
- Composition: f(x) = y, f(y) = x
- Not subtraction
- Not division

Algebraically solve for x,

$$2 \cdot x = 6$$

Algebraically solve for x,

$$2 \cdot x = 6$$

$$(1/2) \cdot 2 \cdot x = (1/2) \cdot 6$$

$$1 \cdot x = 6/2$$

$$x = 3$$

Practice

More properties

- Commutative property: $a \heartsuit b = b \heartsuit a$
- Associative property: $(a \heartsuit b) \heartsuit c = a \heartsuit (b \heartsuit c)$

- (!) commutative changes order of elements
- (!) associative changes order of operations

Distributive property of multiplication and addition

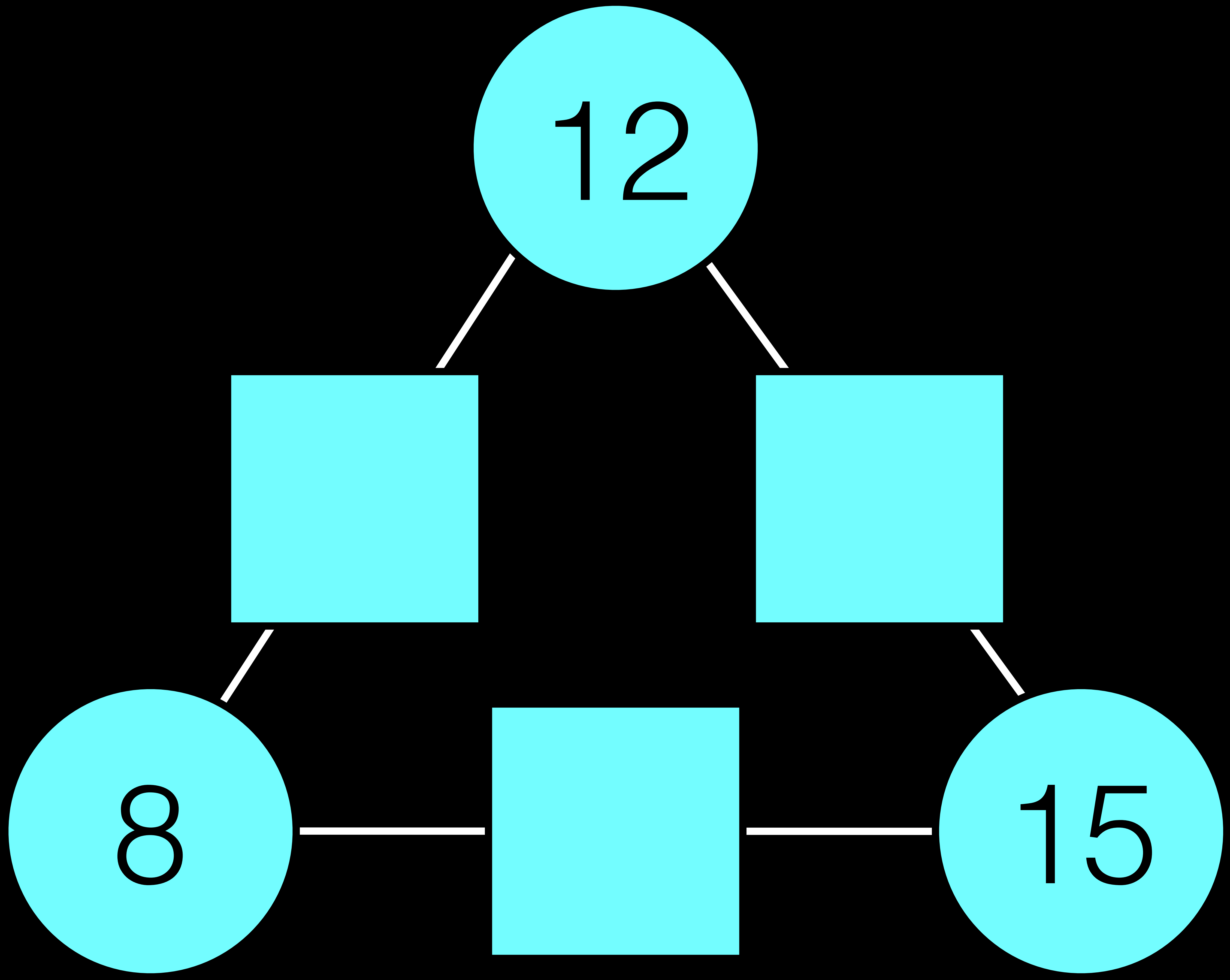
$$a \cdot (b + c) = a \cdot b + a \cdot c$$

- $3 \cdot (x + 2)$

- $3 \cdot x + 3 \cdot 2$

Summary

- Identity element
- Inverse elements
- Commutative property
- Associative property
- (!) Distributive property of multiplicative and addition



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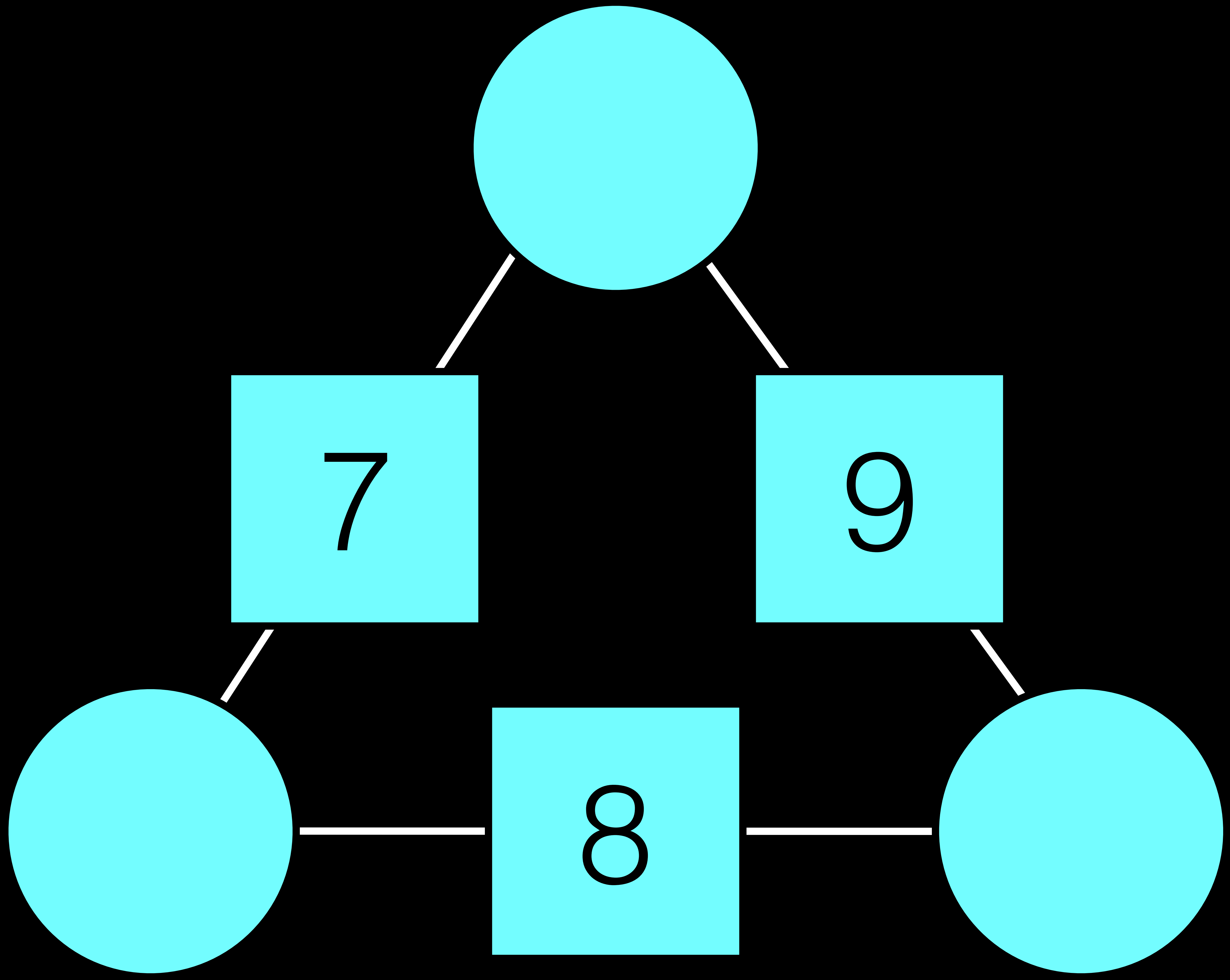

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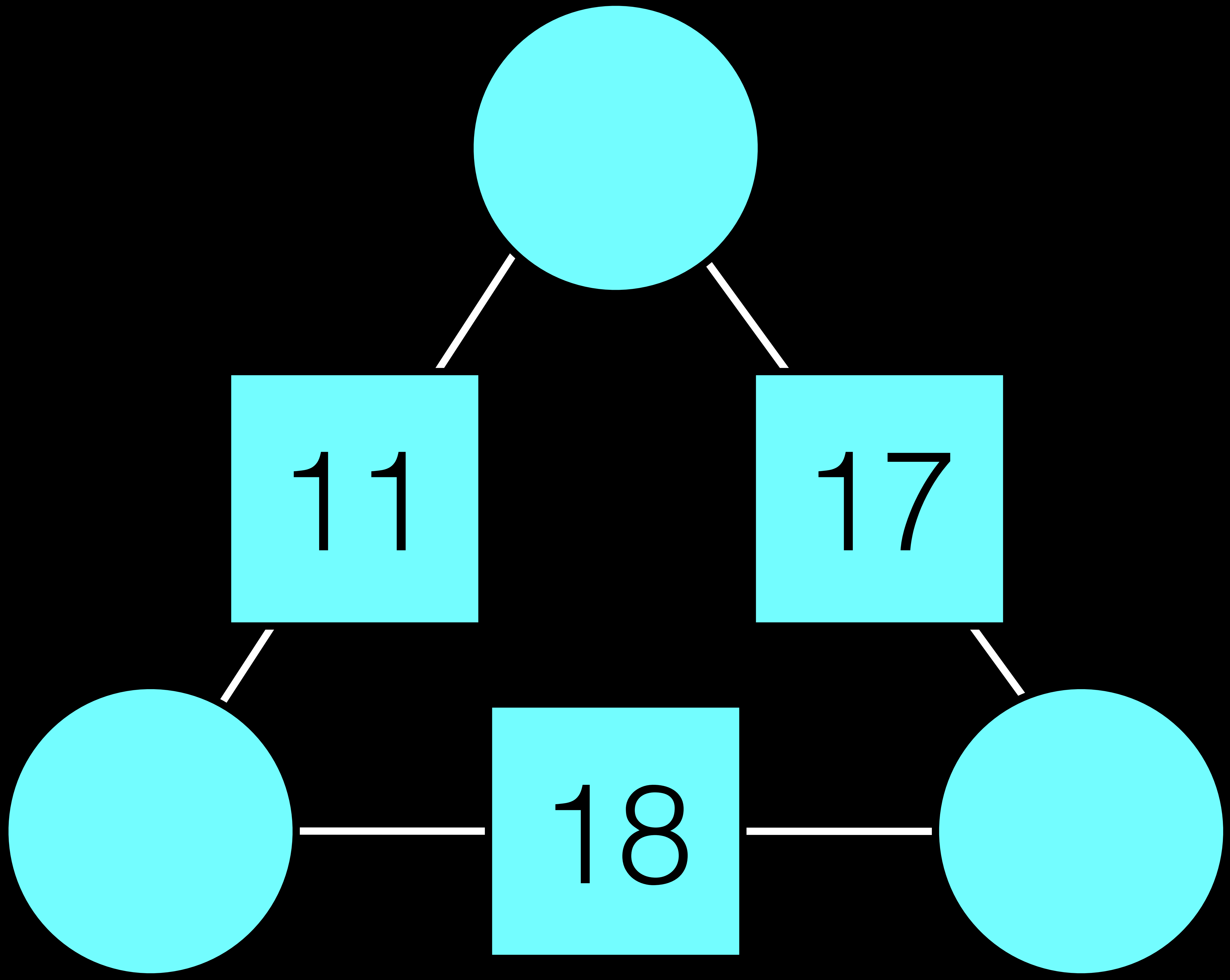

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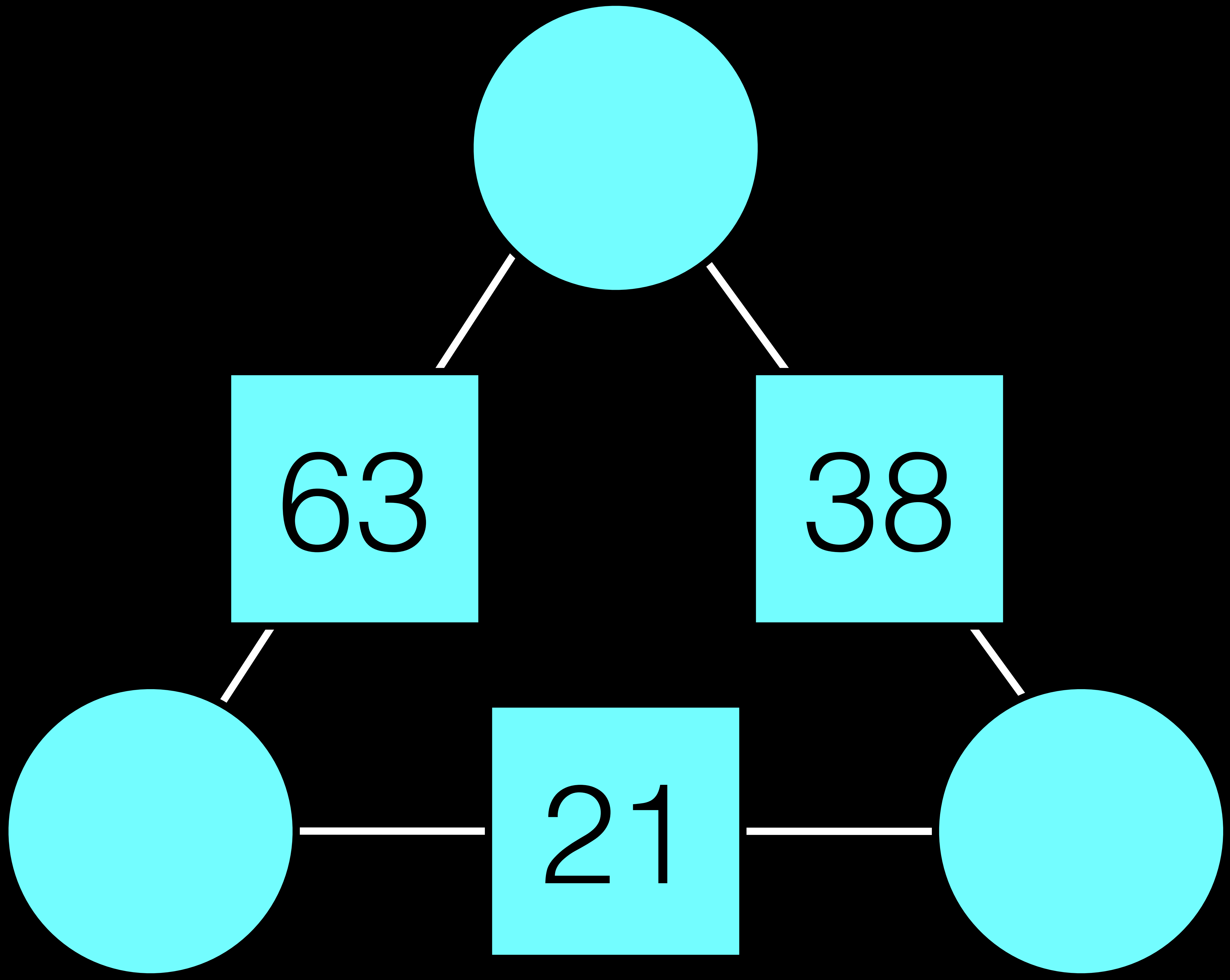

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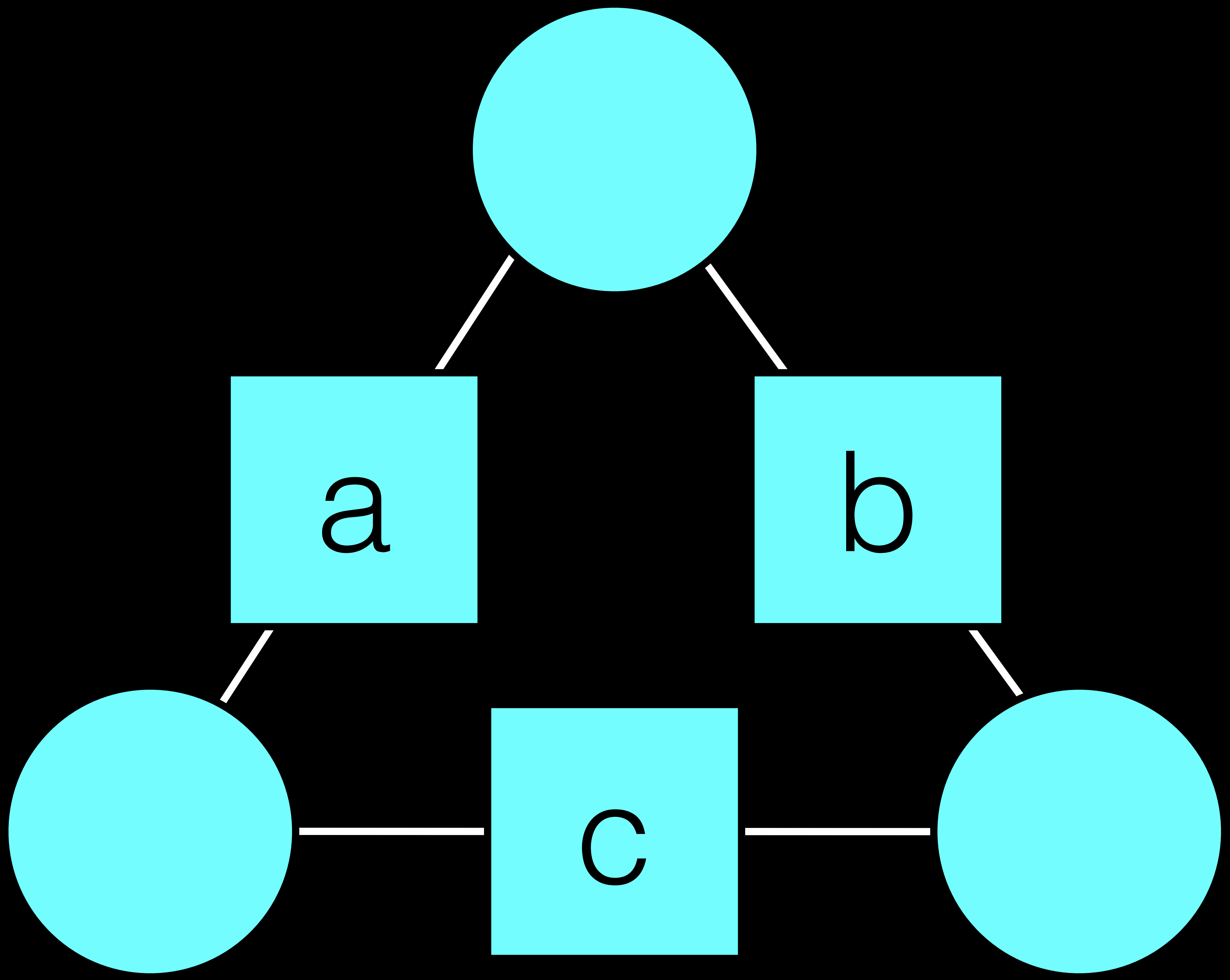

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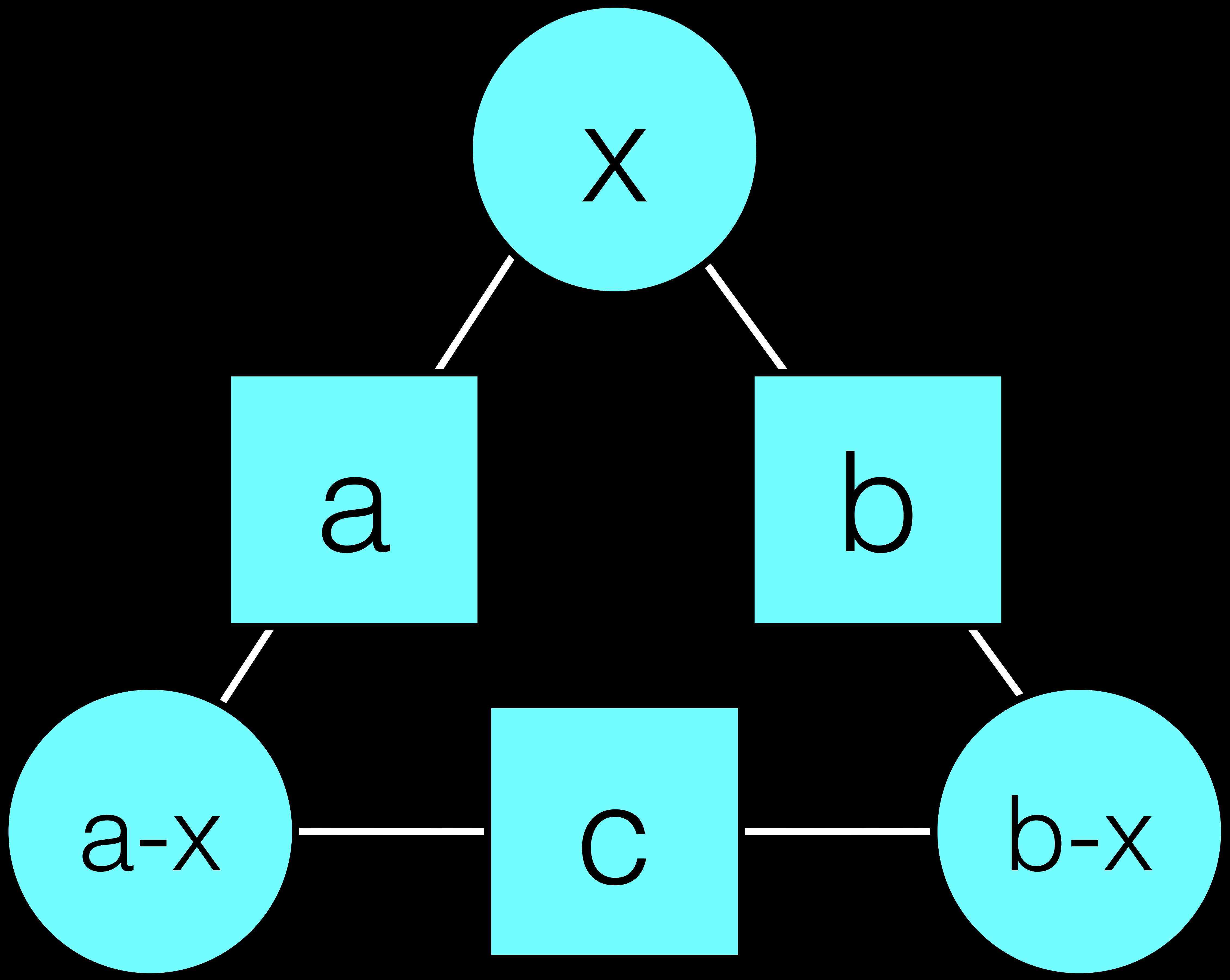

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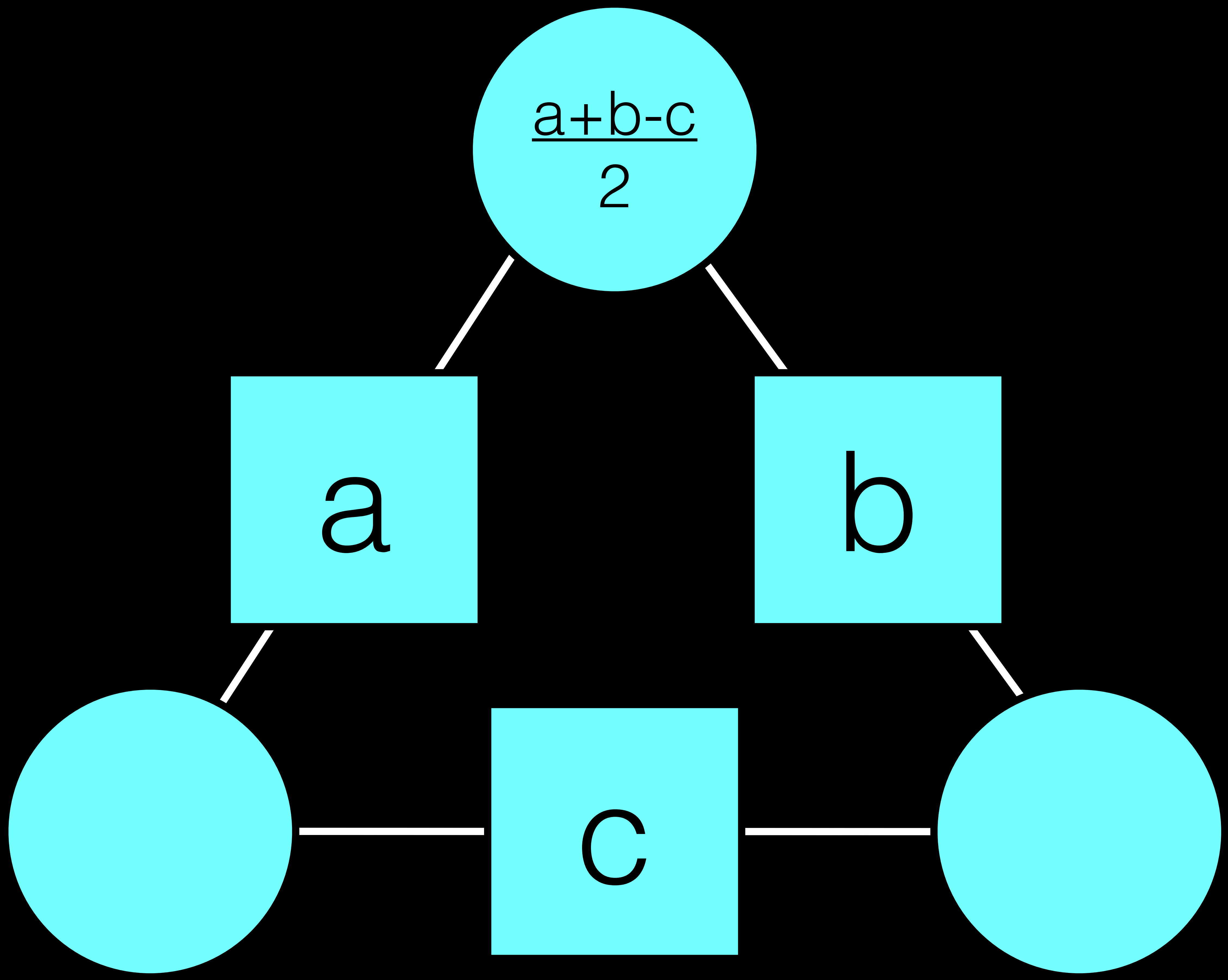

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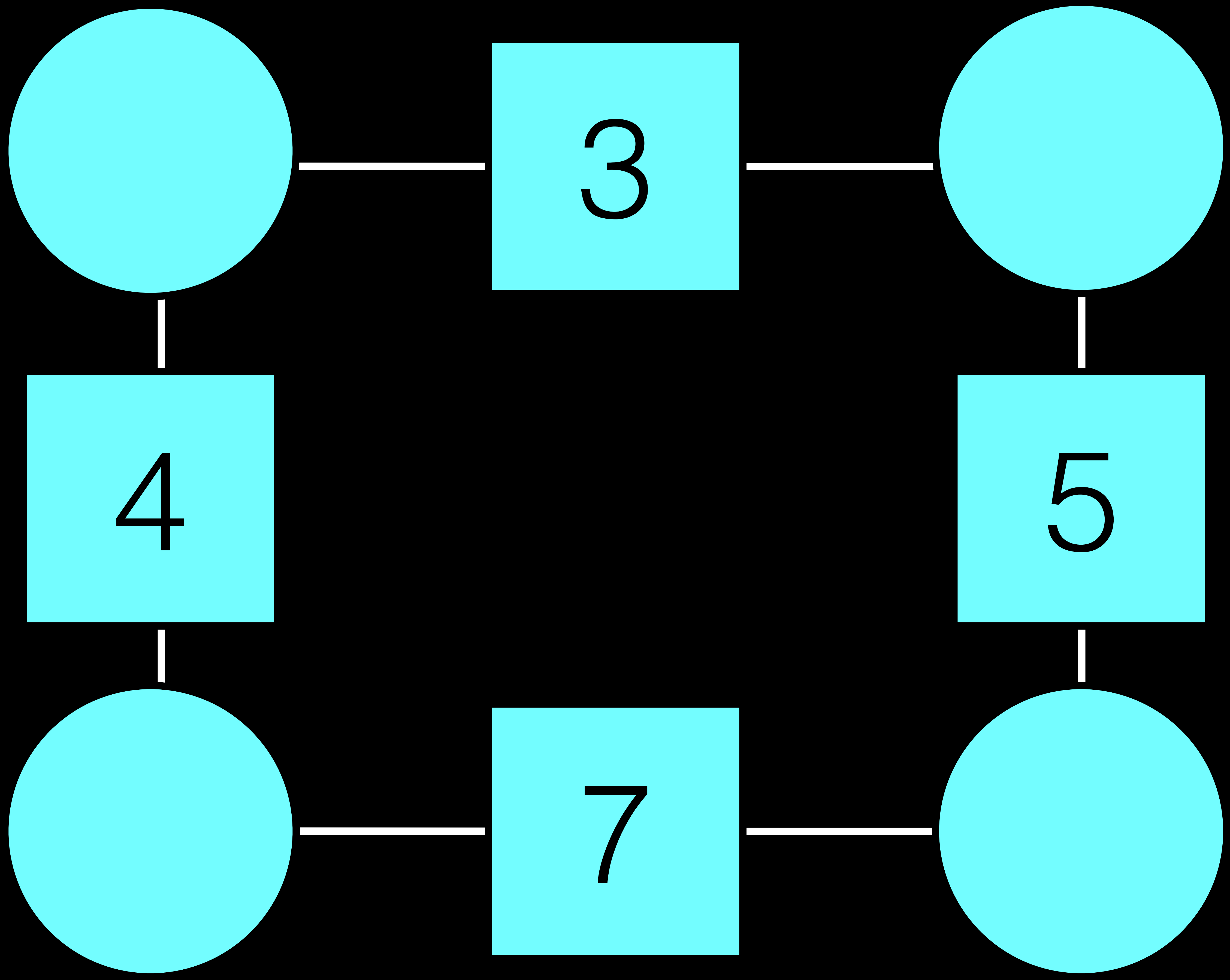

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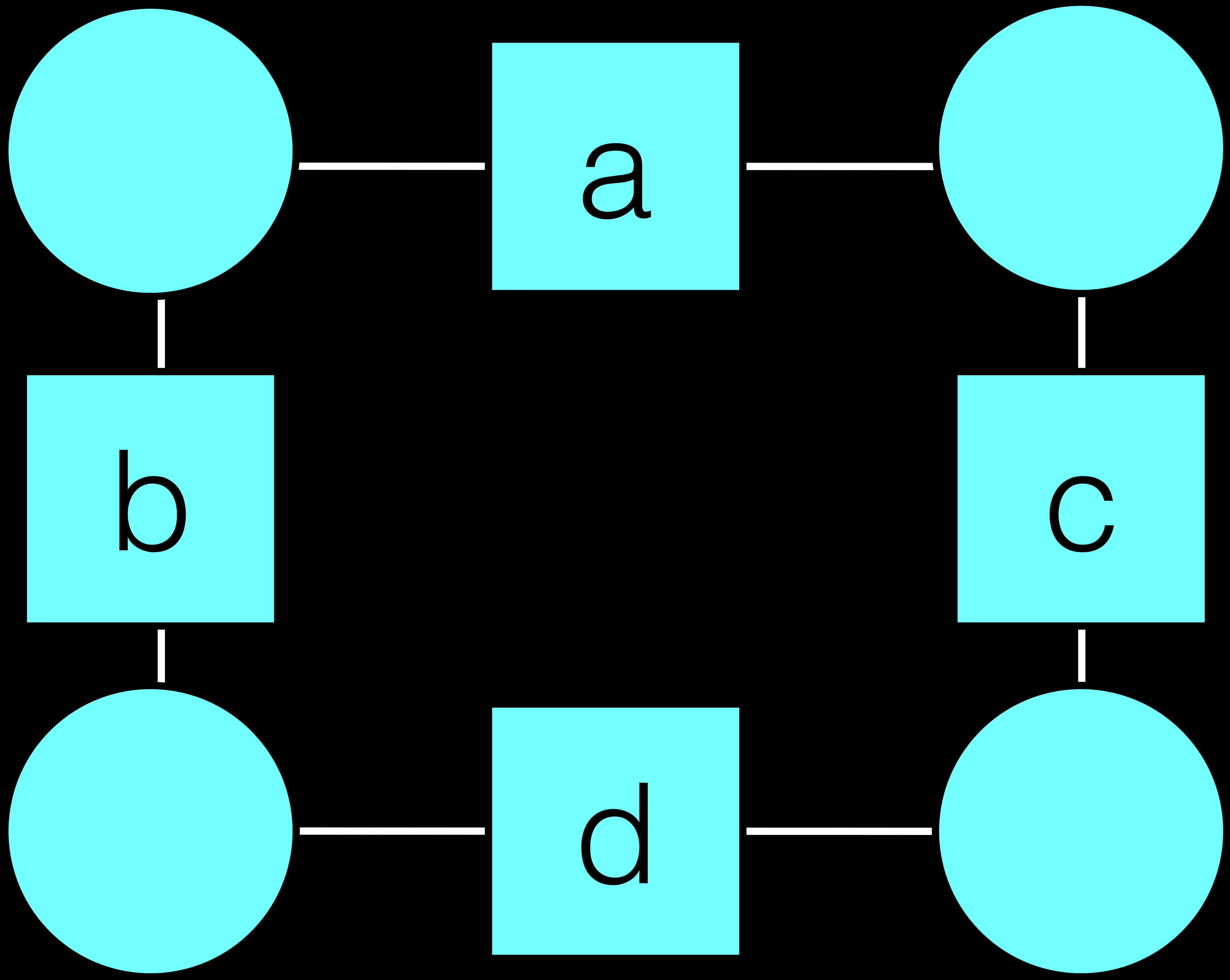

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